intro.mechr Mechanical rotational elements

1 We now introduce a few lumped-parameter elements for mechanical systems in rotational motion. Newton's laws of motion, in their angular analogs, can be applied. Let a torque T and angular velocity Ω be input to a port in a mechanical rotational element. Since, for mechanical rotational systems, the power into the element is

torque

angular velocity

$$\mathcal{P}(t) = T(t)\Omega(t) \tag{1}$$

we call T and Ω the power-flow variables for mechanical rotational systems. Some mechanical rotational elements have two distinct locations between which its angular velocity is defined (e.g. the angular velocity across a spring's two ends) and other elements have just one (e.g. a rotational inertia), the velocity of which must have an inertial frame of reference. This is analogous to how a point in a circuit can be said to have a voltage—by which we mean "relative to ground." In fact, we call this mechanical rotational inertial reference ground.

power–flow variables

ground

2 Work done on the system over the time interval $[0, t_f]$ is defined as

work

$$W \equiv \int_0^{t_f} \mathcal{P}(\tau) d\tau. \tag{2}$$

Therefore, the work done on a mechanical system is

$$W = \int_0^{t_f} T(\tau)\Omega(\tau) d\tau. \tag{3}$$

3 The angular displacement θ is

angular displacement

$$\theta(t) = \int_0^{t} \Omega(\tau) d\tau + \theta(0). \tag{4}$$

Similarly, the angular momentum is

angular momentum

$$h(t) = \int_0^{t} T(\tau) d\tau + h(0). \tag{5}$$

energy storage elements

4 We now consider two elements that can store energy, called energy storage elements; an element that can dissipate energy to a system's environment, called an energy dissipative element; and two elements that can supply power from outside a system, called source elements.

energy dissipative elements

source elements

Rotational springs

5 A rotational spring is defined as an element for which the angular displacement θ across it is a monotonic function of the torque T through it. A linear rotational spring is a rotational spring for which the angular form of Hooke's law applies; that is, for which

rotational spring

linear rotational spring

$$T(t) = k\theta(t), \tag{6}$$

where T is the torque through the spring and θ is the angular displacement across the spring and k is called the torsional spring constant and is typically a function of the material properties and geometry of the element. This is called the element's constitutive equation because it constitutes what it means to be a rotational spring.

torsional spring constant

constitutive equation

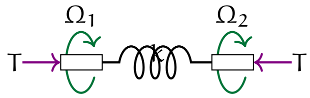

Figure mechr.1: schematic symbol for a spring with spring constant k and angular velocity drop $\Omega = \Omega_1 - \Omega_2$.

6 Although there are many examples of nonlinear springs, we can often use a linear model for analysis in some operating regime. The elemental equation for a linear spring can be found by time-differentiating Equation 6 to obtain

elemental equation

We call this the elemental equation because it relates the element's power-flow variables T and Ω.

7 A rotational spring stores energy as elastic potential energy, making it an energy storage element. The amount of energy it stores

depends on the torque it applies. For a linear rotational spring,

$$\mathcal{E}(t) = \frac{1}{2k} T(t)^2. \tag{7}$$

Moments of inertia

moment of inertia

8 A moment of inertia element with moment of inertia J, angular velocity Ω (relative to an inertial reference frame), and angular momentum h has the constitutive equation

$$h = J\Omega. \tag{8}$$

Once again, time-differentiating the constitutive equation gives us the elemental equation:

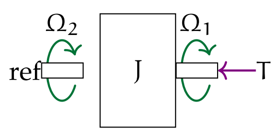

Figure mechr.2: schematic symbol for a moment of inertia with inertia J and velocity drop $\Omega = \Omega_1 - \Omega_2$, where Ω_2 is a constant reference velocity.

which is just the angular version of Newton's second law.

9 Any rotating element with mass can be considered as a lumped-inertia element. The flywheel is the quintessential example. Flywheels store energy in their angular momentum, with the expression

flywheel

$$\mathcal{E}(t) = \frac{1}{2} J\Omega^2, \tag{9}$$

making them (and all moments of inertia) energy storage elements.

Rotational dampers

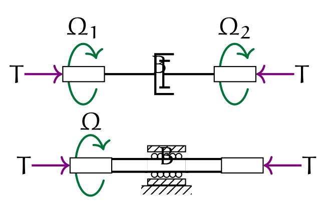

Figure mechr.3: schematic symbol for a drag cup (above) and bearing (below) with damping coefficient B. For the drag cup, the angular velocity drop is $\Omega = \Omega_1 - \Omega_2$ and for the bearing, Ω is reference is ground.

10 Rotational dampers are defined as elements for which the torque T through the element is a monotonic function of the angular velocity Ω across it. Linear rotational dampers have constitutive equation (and, it turns out, elemental equation)

rotational dampers
linear rotational dampers

$$T = B\Omega \tag{10}$$

where B is called the torsional damping

torsional damping coefficient
torsional viscous damping

coefficient. Linear torsional damping is often called torsional viscous damping because systems that push viscous fluid through small orifices or those that have lubricated bearings are well-approximated by this equation. For this reason, a damper is typically schematically depicted as a drag cup or as a bearing, both of which are shown in Figure mechr.3.

drag cup
bearing

11 Linear damping is a reasonable approximation of lubricated sliding, but it is rather poor for dry friction or Coulomb friction, forces for which are not very velocity-dependent.

dry friction
Coulomb friction

12 Rotational dampers dissipate energy from the system (typically to heat), making them energy dissipative elements.

Torque and angular velocity sources

13 An ideal torque source is an element that provides arbitrary energy to a system via an independent (of the system) torque. The corresponding angular velocity across the element depends on the system.

ideal torque source

14 An ideal angular velocity source is an element that provides arbitrary energy to a system via an independent (of the system) angular velocity. The corresponding torque through the element depends on the system.

ideal angular velocity source