# emech.trans Ideal transducers

1 Two-port system elements can model transducers—elements that transfer energy between two energy domains or change its form within an energy domain. The quintessential example, which we will consider in detail, is the motor, which converts electrical energy to mechanical energy. However, many other system elements can be considered transducers, and we'll consider a few in this lecture.

**two−port elements**

**transducers**

**motor**

2 Each of the two ports has a through- and an across-variable. We use the convention that the power into each port ($\mathcal{P}_1$ and $\mathcal{P}_2$) is positive, which has implications for the signs of the power flow variables $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{V}_1$, and $\mathcal{V}_2$. For an two-port element to transfer power, we have

We define the transformer ratio TF to be

$$\mathrm{TF} \equiv \frac{\mathcal{V}_1}{\mathcal{V}_2} = -\frac{\mathcal{F}_2}{\mathcal{F}_1}. \tag{1}$$

**transformer ratio**

Furthermore, we define the gyrator modulus GY to be

$$\mathrm{GY} \equiv \frac{\mathcal{V}_1}{\mathcal{F}_2} = -\frac{\mathcal{V}_2}{\mathcal{F}_1}. \tag{2}$$

**gyrator modulus**

3 For an ideal transducer—one that is linear, time-invariant, and without power loss—we have only two nontrivial solutions:[1]

**ideal transducer**

$$\mathcal{V}_2 = \mathcal{V}_1/\mathrm{TF} \quad \text{or} \quad \mathcal{V}_2 = -\mathrm{GY}\,\mathcal{F}_1$$
$$\mathcal{F}_2 = -\mathrm{TF}\,\mathcal{F}_1 \qquad \mathcal{F}_2 = \mathcal{V}_1/\mathrm{GY}.$$

1. For an explanation of why that is the case, see Rowell and Wormley. (Derek Rowell and David N. Wormley. System Dynamics: An Introduction. Prentice Hall, 1997)

4 For a given element, if the solution with TF is a good model, we call that element a transformer. If the GY solution is a good model, we call it a gyrator.

**transformer**

**gyrator**

**Example emech.trans–1** **re: DC motor**

Consider a DC motor with rotor radius $r$, number of coil turns $N$, background field $B$, and rotor length $\ell$. The torque $T$ of a DC motor is related to its coil current $i$ by the relation

$$T = -2rNB\ell i.$$

1. Determine if DC motors are transformers or gyrators.
2. Find TF or GY.
3. Derive the relation between the voltage $v$ and the angular velocity $\Omega$ across the motor using the assumption that it is an ideal transducer.

**Example emech.trans–2** **re: gears**

Consider two gears with radii $r_1$ and $r_2$ and number of teeth $n_1$ and $n_2$.

1. Determine the power flow variables for gears.
2. Write two independent equations relating the power flow variables.
3. Determine if gears are transformers or gyrators.
4. Find TF or GY.