Table real.1: datasheet specifications for the Electrocraft 23SMDC-LCSS servomotor from Servo Systems. This is the motor used in the lab.

parameter	specification	SI conversion
continuous stall torque	55 oz-in	0.388 N-m
peak torque T_{max}	400 oz-in	2.82 N-m
max terminal voltage	60 V_{dc}	60 V_{dc}
max operating speed Ω_{max}	6000 rpm	628 rad/s
rotor inertia J_m	0.008 oz-in/s^2	$56.5 \cdot 10^{-6}$ N-m/s^2
damping constant B_m	0.25 oz-in/krpm	$16.9 \cdot 10^{-6}$ N-m/(rad/s)
thermal resistance	4 C/W	4 K/W
max armature temp	155 C	428 K
max friction torque	3 oz-in	0.0212 N-m
max radial load[7]	10 lb	44.5 N
weight (motor only)	3.5 lb	15.6 N
torque constant K_t	13.7 oz-in/A	0.097 N-m/A
voltage constant K_v	10.2 V/krpm	0.097 V/(rad/s)
terminal resistance	1.6 Ω	1.6 Ω
electrical time constant	2.6 ms	$2.6 \cdot 10^{-3}$ s
mechanical time constant	8.9 ms	$8.9 \cdot 10^{-3}$ s
max continuous current	4 A	4 A
armature inductance	4.1 mH	$4.1 \cdot 10^{-3}$ H
max peak current	34 A	34 A

emech.real Modeling a real electromechanical system

1 We now model the electromechanical systems from the laboratory, shown in Figure real.1. The system includes a brushed DC motor (Electrocraft 23SMDC-LCSS servomotor from Servo Systems), two shafts, a shaft coupler, two bearings, and a flywheel. The motor's datasheet specifications are given in Table real.1. The mechanical subsystem's inertia is dominated by the stainless steel flywheel with $J_f = 0.324 \cdot 10^{-3}$ kg-m^2. The bearing damping B_b is the most difficult parameter to determine. Let's begin with the assumption that the combined bearing damping is $B_b = 20 \cdot 10^{-6}$ N-m/(rad/s).

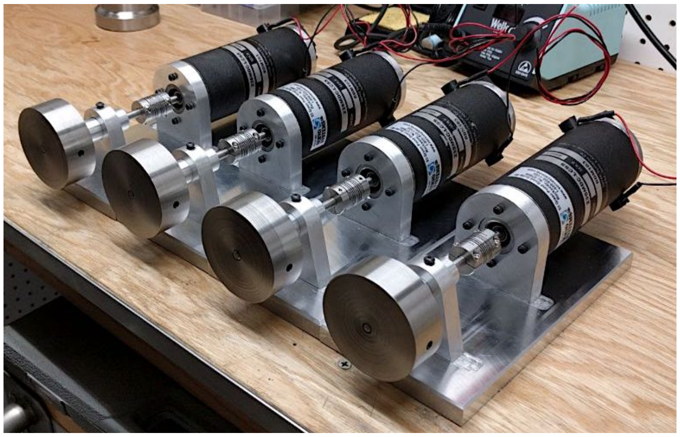

Figure real.1: electromechanical systems from the lab.

7. Load applied at one inch from bearing.

Linear graph model

2 A linear graph model is in order. An ideal

voltage source drives the motor[8]—modeled as an ideal transducer with armature resistance R and inductance L, given in Table real.1. The ideal transducer's rotational mechanical side (2) is connected to a moment of inertia $J = J_m + J_f = 0.381 \cdot 10^{-3}$ kg-m^2, dominated by the flywheel,[9] and damping B, which is the parallel combination of the internal motor damping of Table real.1 and the bearing damping B_b, to yield $B = 26.9 \cdot 10^{-6}$ N-m/s^2. We choose to ignore the flexibility of the coupler. Problem emech. considers the same system but does not ignore the coupler's flexibility. In general, shaft couplers have significant flexibility and, depending on the application, this may require consideration in the dynamic model.

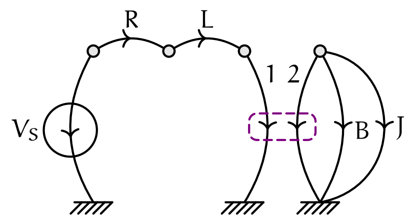

Figure real.2: a linear graph model of the electromechanical systems of Figure real.1.

8. Often we can model our motor-driving source as ideal within an operating range. See Lecture emech.drive for more details.

9. This is the sum of the inertia of the flywheel $J_f = 0.324 \cdot 10^{-3}$ kg-m^2 and the rotor $J_m = 0.0565 \cdot 10^{-3}$ kg-m^2. It might be worthwhile combining this with the inertia from the shaft and coupler to obtain a more accurate value, but the difference is likely negligible.

State-space model

3 The normal tree can be constructed by the procedure from Lecture emech.transmod. The voltage source V_S is first included, followed by J. Then exactly one edge of the ideal transducer must be selected, minimizing the number of T-types in the tree. We don't really have a choice, in this case, because selecting edge 2 would create a loop, so we must select edge 1. Next, R is included. No more elements can be included without creating a loop, so we are finished.

4 We are now prepared to determine variables. The state variables are across variables of A-type tree branches and through variables of T-type links—so Ω_J and i_L, and the system is second-order ($n = 2$). Clearly, the system's input is the voltage source V_S. We are interested in all the variables for the analysis in Lecture emech.curves, so we choose them all for our outputs. In summary, then, the state, input,

and output vectors are:

$$x = \begin{bmatrix} \Omega_J \\ i_L \end{bmatrix}, \; u = \begin{bmatrix} V_s \end{bmatrix}, \text{ and}$$

$$y = \begin{bmatrix} \Omega_J & T_J & v_L & i_L & \Omega_B & T_B & v_R & i_R & v_1 & i_1 & \Omega_2 & T_2 & V_s & I_s \end{bmatrix}^\top .$$

5 Let's write some equations! Elemental are up first.

J		R	
L		1	
B		2	

Now, continuity and compatibility equations are developed by summing through-variables into contours. The three required contours—one for each of R, 1, and J—can be drawn on Figure real.3. The three compatibility equations—one for each of L, 2, and B—are found by "temporarily including" those links in the tree and summing across-variables around the loops created. Let's write the equations.

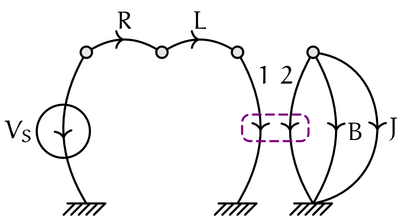

Figure real.3: the linear graph model for drawing contours.

branch	continuity equation
R	
1	
J	

link	compatibility equation
L	
2	
B	

6 All that remains to form the state-space model is to eliminate variables that are neither states nor inputs from the elemental, continuity, and compatibility equations. Eliminating secondary variables by substituting the continuity and compatibility equations into the elemental equations, the following results.

J		R
L		1
B		2

The last four equations allow us to eliminate the remaining undesirable variables to obtain the state model in the standard form[10]

$$\frac{dx}{dt} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

where

$$A = \begin{bmatrix} -B/J & TF/J \\ -TF/L & -R/L \end{bmatrix}, \tag{1c}$$

$$B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \tag{1d}$$

$$C = \begin{bmatrix} 1 & -B & -TF & 0 & 1 & B & 0 & 0 & TF & 0 & 1 & 0 & 0 & 0 \\ 0 & TF & -R & 1 & 0 & 0 & R & 1 & 0 & 1 & 0 & -TF & 0 & 1 \end{bmatrix}^\top, \text{ and} \tag{1e}$$

$$D = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^\top. \tag{1f}$$

10. Here is the `rnd` file for use with StateMint (`statemint.stmartin.edu`) to derive the state-space model from the elemental, continuity, and compatibility equations:

`ricopic.one/dynamic_systems/source/motor_model.rnd`

Note that the “constraint equations” are the continuity and compatibility equations solved for primary variables.