

## emech.exe Exercises for Chapter emech

### Exercise emech.triangle

Respond to the following questions and imperatives with one or two sentences and, if needed, equations and/or sketch.

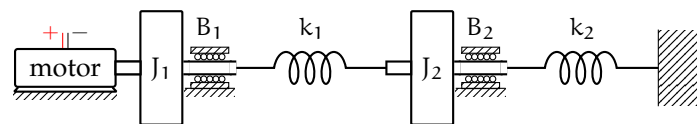
- Why do we include a resistor in lumped-parameter motor models?
- How are brushes used in brushed DC motors?
- With regard to standard motor curves, why do we say the “braking power” is equivalent to the power that could be successfully transferred by the motor to the mechanical system?
- In terms of electrical and mechanical processes, why does an efficiency versus torque motor curve have a peak?
- As a DC motor’s bearings wear down, how will its efficiency curve be affected?

### Exercise emech.square

Consider the system presented in the schematic of Fig. exe.1. Let the DC motor have motor constant  $K_a$  (units N-m/A) and let the motor be driven by an ideal current source  $I_s$ . Assume the motor inertia has been lumped into  $J_1$  and motor damping lumped into  $B_1$ .

- Draw a linear graph model.
- Draw a normal tree.
- Identify any dependent energy storage elements. If the motor was driven by an ideal voltage source instead, how would this change?

Draw a linear graph model and normal tree.



**Figure exe.1:** schematic of an electromechanical system for Exercise emech..

Exercise emech.rectangle

Consider the system presented in the schematic of Fig. exe.1. From the linear graph model and normal tree derived in Exercise emech., derive a state-space model in standard form. Let the outputs be  $\theta_{J_1}$  and  $\theta_{J_2}$ , the angular positions of the flywheels.

Exercise emech.quadrilateral

Consider the linear graph model of a motor coupled to a rotational mechanical system shown in Fig. exe.2. This is similar to the model from the Lec. emech.real, but includes the flexibility of the shaft coupler. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance  $R$  and inductance  $L$ , given by the manufacturer in Table real.1. The ideal transducer’s rotational mechanical side (2) is connected to a moment of inertia  $J_m$  modeling the rotor inertia and damping  $B_b$  modeling the internal motor damping, both values given in the motor specifications. Take  $B_b = B_m$  and  $J_f = 0.324 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$ . Assume the shaft coupling has a torsional stiffness of  $k = 100 \text{ N}\cdot\text{m}/\text{rad}$ .

- Derive a state-space model for the system with outputs  $i_1$  and  $\Omega_{J_f}$ .
- Create a Matlab ss model for the system and simulate its response from rest to an input voltage  $V_S = 10 \text{ V}$ .
- Plot the outputs through time until they reach steady state.

Exercise emech.mrpotatohead

Consider the linear graph model (with normal tree) of Fig. exe.3. This is a model of a motor with constant  $K_a$  connected to a pair of meshing

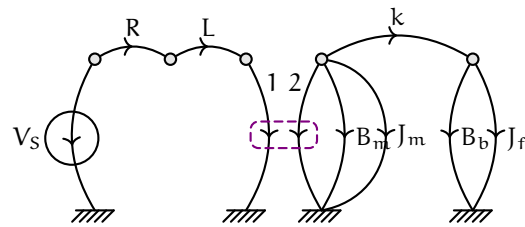


Figure exe.2: a linear graph model of the electromechanical system.

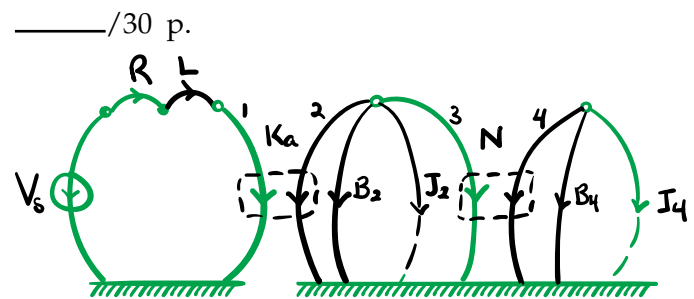


Figure exe.3: a linear graph model with normal tree in green of the electromechanical system of Exercise emech. with a gear reduction.

gears with transformer ratio  $N$ , the output over input gear ratio. An ideal voltage source drives the motor—modeled as an ideal transducer with armature resistance  $R$  and inductance  $L$ . The motor’s rotational mechanical side (2) is connected to a moment of inertia  $J_2$  modeling the rotor and drive gear combined inertia. The damping element  $B_2$  models the internal motor damping and the drive gear bearing damping. The output side of the gear transducer (4) is connected to a moment of inertia  $J_4$  modeling the output gear and load combined inertia. The damping element  $B_4$  models the internal motor damping and the drive gear bearing damping. Use the parameter values given in Table exe.1.

- Derive a state-space model for the system with outputs  $\Omega_{J_2}$  and  $\Omega_{J_4}$ .
- Create a Matlab ss model for the system and simulate its response from rest to an input voltage  $V_s = 20$  V.
- Plot the outputs through time until they reach steady state.

Exercise emech.clunker

Draw a linear graph, a normal tree, identify state variables, identify system order, and denote any dependent energy storage elements for each of the following schematics.

- The electronic system of Fig. exe.4, voltage and current sources, and transformer with transformer ratio  $N$ .
- The electromechanical system of Fig. exe.5 with motor model parameters shown, coordinate arrow in green. Model the propeller as a moment of inertia  $J_2$  and damping  $B_2$ .
- The translational mechanical system of Fig. exe.6, force source, coordinate arrow in green.

Table exe.1: parameter values for Exercise emech..

$R$	$2 \Omega$
$L$	$8 \text{ mH}$
$K_a$	$0.2 \text{ N}\cdot\text{m}/\text{A}$
$J_2$	$0.1 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$
$B_2$	$50 \mu\text{N}\cdot\text{m}/(\text{rad}/\text{s})$
$N$	$5$
$J_4$	$1 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$
$B_4$	$70 \mu\text{N}\cdot\text{m}/(\text{rad}/\text{s})$

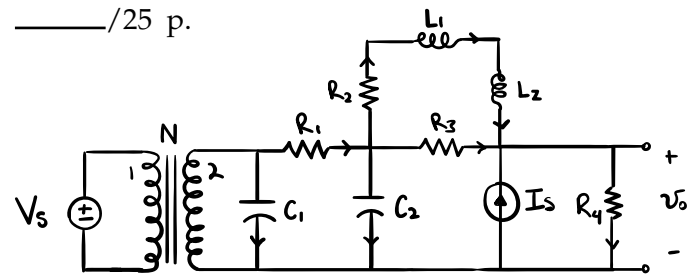


Figure exe.4: Exercise emech. part (a)

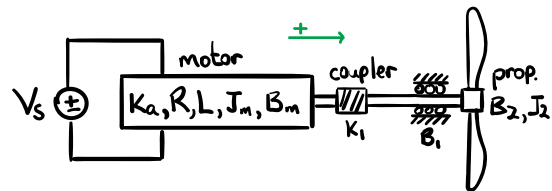


Figure exe.5: Exercise emech. part (b)

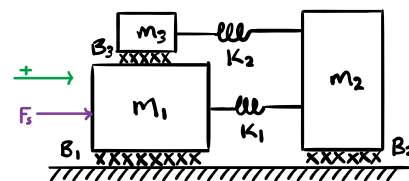


Figure exe.6: Exercise emech. part (c)

Exercise emech.curvy

Consider the DC motor curves of Fig. curves.2, ———/20 p.  
reproduced in Fig. exe.7.

- At peak efficiency, what is the steady-state motor speed?
- At peak efficiency, what is the steady-state motor torque?
- You are to use this motor to drive a load at a constant angular speed of 100 rad/s with at least 1 N-m of torque. You wisely choose to use a gear reduction between the motor and load. What should the gear ratio be to meet the above requirements and optimize efficiency? Justify your answer in terms of the motor curves of Fig. exe.7.

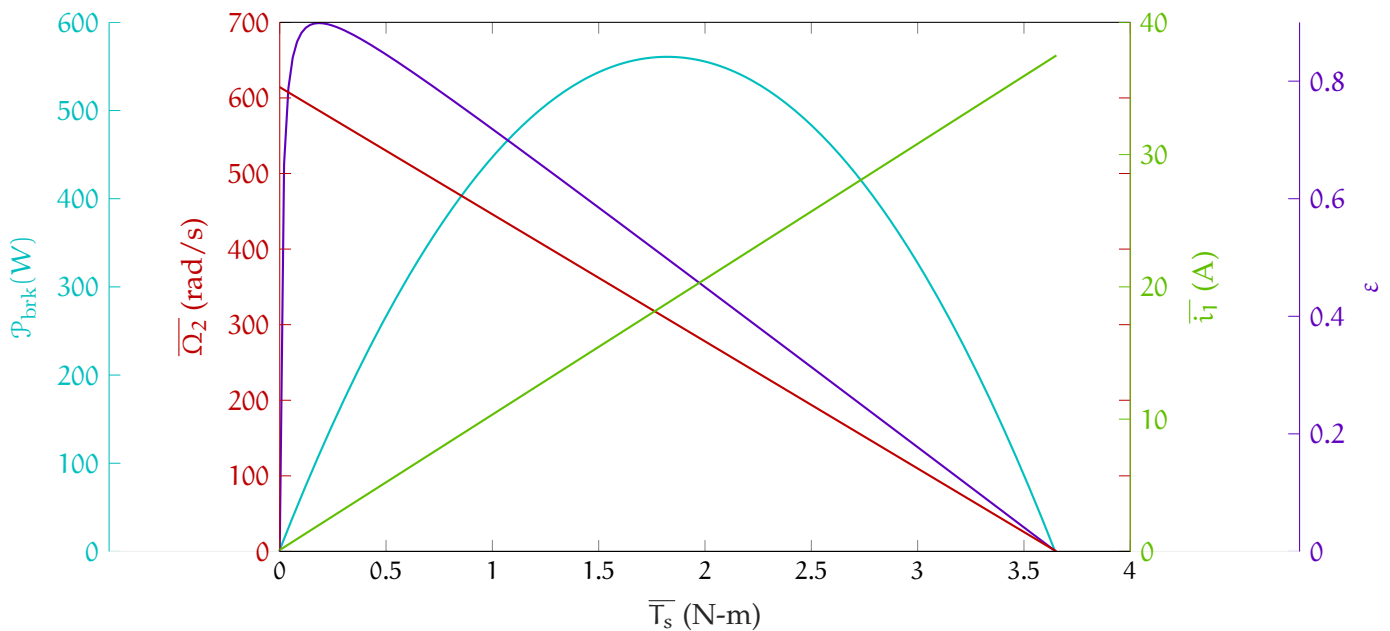


Figure exe.7: the motor curve Fig. curves.2 for Exercise emech..

Exercise emech.chair

Consider the opamp circuit of Fig. exe.8, which ———/25 p.  
will be used to drive a motor. The input can

supply a variable  $V_S \in [0, 10]$  V, the motor has constant  $K_a = 0.05$  V/(rad/s) and coil resistance  $R_m = 1 \Omega$ , and the opamp has differential supplies  $\pm 24$  V. Assume the maximum torque magnitude required from the motor at top speed is  $|T_2| = 0.1$  N-m and ignore any voltage drop in the motor due to the coil inductance.<sup>14</sup> Select  $R_1$  and  $R_2$  to demonstrably meet the following requirements:

- a. drivable motor speeds of at least  $[0, 400]$  rad/s,
- b. no saturation of the opamp, and
- c. a maximum power dissipation by  $R_1$  and  $R_2$  less than 300 mW.

Exercise emech.onomatopoeia

Consider the DC motor curves of Fig. curves.2, reproduced in Fig. exe.7. If this motor is running at  $400 \frac{\text{rad}}{\text{s}}$ ,

- 1. How much torque is produced?
- 2. What is the output power?
- 3. What is the input power?
- 4. Why are the input and output power the same or different?

Exercise emech.deglazification

Explain in your own words what lumped parameter elements should be used when modeling an electric motor and why.

Exercise emech.confuzzled

In the linear graph below a system is depicted consisting of a motor with its related damping and inertia driven by a voltage source and connected to a set of gears driving a second inertia. A rotary spring is attached between the two inertias.

14. Do not ignore the voltage drop across  $R_m$ , though. Note that this amounts to an assumption of steady-state operation at top speed. By requiring a specific  $T_2$ , we are also implicitly ignoring torque losses due to motor bearing damping.

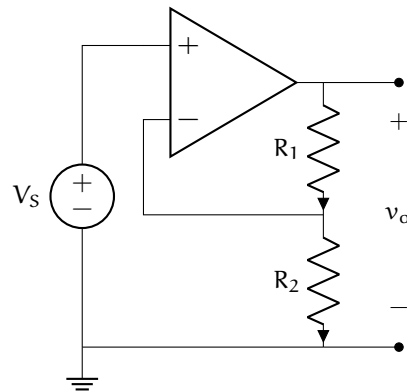


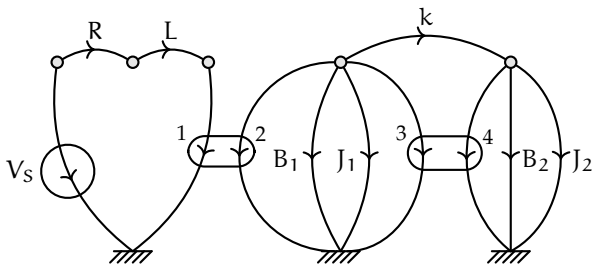
Figure exe.8: opamp circuit for Exercise emech..

\_\_\_\_\_/10 p.

\_\_\_\_\_/20 p.

Given this linear graph:

1. draw a normal tree,
2. determine the state variables and system order, and
3. list any dependent energy storage elements and explain what this implies.



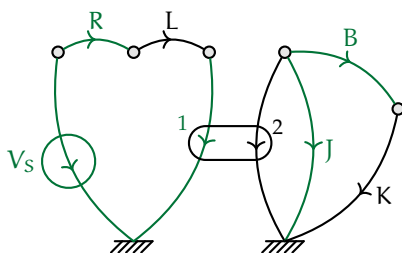
Exercise emech.levitation

—/40 p.

In the linear graph and normal tree below a system is depicted consisting of a motor driven by a voltage source  $V_s$  with inertia  $J$  driving a rotary damper and spring connected in series. Let the motor constant be  $K_a$ , and outputs of the system be the rotational velocity of the inertia,  $\Omega_J$ , and the change in rotational velocity across the rotational damper,  $\Omega_B$ .

Given this linear graph and normal tree:

1. determine the state variables,
2. define the state, input, and output vectors,
3. write the elemental, continuity, and compatibility equations, and
4. solve for the state and output equations.



## **Part II**

### **Time response**

## Linear time–invariant system properties

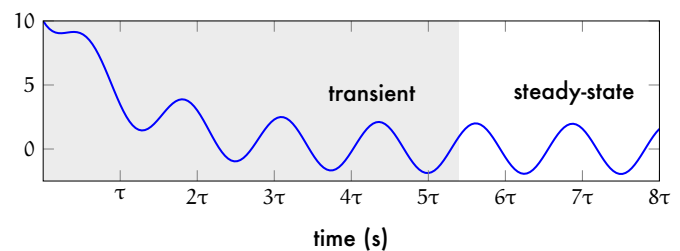
1 In this chapter, we will extend our understanding of linear, time-invariant (LTI) system properties. We must keep in mind a few important definitions.

2 The transient response of a system is its response during the initial time-interval during which the initial conditions dominate. The steady-state response of a system is its remaining response, which occurs after the transient response. Fig. lti.1 illustrates these definitions.

3 The free response of a system is the response of the system to initial conditions—without forcing (i.e. the specific solution of the io ODE with the forcing function identically zero). This is closely related to, but distinct from, the transient response, which is the free response plus an additional term. This additional term is the forced response: the response of the system to a forcing function—without initial conditions (i.e. the specific solution of the io ODE with the initial conditions identically zero).

transient response

steady–state response



**Figure lti.1:** transient and steady-state responses. Note that the transition is not precisely defined. (Figure adapted from Electronics: an introduction by Picone.)

free response

forced response