### ssresp.exe Exercises for Chapter ssresp

Exercise ssresp.larry

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

- a. Find the eigenvalue matrix  $\Lambda$  and comment on the stability of the system (justify your comment). Use the convention that  $\lambda_1 \leq \lambda_2$  and order  $\Lambda$ accordingly.
- b. Find the eigenvectors and the modal matrix M.
- c. Find the state transition matrix  $\Phi(t)$ . Hint: first find the "diagonalized" state transition matrix  $\Phi'(t)$ .
- d. Using the state transition matrix, find the output homogeneous solution for initial condition

$$\mathbf{x}(\mathbf{0}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Exercise ssresp.mo

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the eigenvalue matrix  $\Lambda$  and comment on the stability of the system (justify your comment). Use the convention that  $\lambda_1 \leq \lambda_2$  and order  $\Lambda$ accordingly.

- b. Find the eigenvectors and the modal matrix M.
- c. Find the state transition matrix  $\Phi(t)$ . Hint: first find the "diagonalized" state transition matrix  $\Phi'(t)$ .
- d. Using the state transition matrix, find the output homogeneous solution for initial condition

$$\mathbf{x}(\mathbf{0}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}.$$

Exercise ssresp.curly

Use a computer for this exercise. Let a system have the following state A-matrix:

$$A = \begin{bmatrix} -2 & 2 & 0 \\ -1 & -2 & 2 \\ 0 & -1 & -2 \end{bmatrix}.$$

For this system, answer the following imperatives.

- a. Find the eigenvalue matrix  $\Lambda$  and modal matrix M.
- b. Comment on the stability of the system (justify your comment).
- c. Find the diagonalized state transition matrix  $\Phi'(t)$ . Be sure to print the expression. Furthermore, find the state transition matrix  $\Phi(t)$ .
- d. Using the state transition matrix, find the state free response for initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Do not print this expression.

e. Plot the free response found above for

 $t \in [0, 4]$  seconds.

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Exercise ssresp.lonely

Use a computer for this exercise. Let a system have the following state and output equation matrices:

 $A = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \end{bmatrix}.$ 

For this system, answer the following imperatives.

- a. Find the eigenvalue matrix  $\Lambda$  and comment on the stability of the system (justify your comment). Use the convention that  $\lambda_1 \ge \lambda_2 \ge \lambda_3$  and order  $\Lambda$ accordingly.
- b. Find the eigenvectors and the modal matrix *M*.
- c. Find the state transition matrix  $\Phi(t)$ . Hint: first find the "diagonalized" state transition matrix  $\Phi'(t)$ .
- d. Let the input be

$$\mathbf{u}(\mathbf{t}) = \begin{bmatrix} 4\\ \sin(2\pi\mathbf{t}) \end{bmatrix}.$$

Solve for the forced state response  $x_{fo}(t)$ . Express it simply—it's not that bad.

- e. Solve for the forced output response  $y_{fo}(t)$ . Express it simply—it's not that bad.
- f. Plot  $\mathbf{y}_{fo}(t)$  for  $t \in [0, 7]$  sec.

Exercise ssresp.argentina

Given a state space system,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
  
 $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u},$ 

with,

$$A = \begin{bmatrix} -8 & -6 \\ 3 & 1 \end{bmatrix} \text{ and}$$
$$C = \begin{bmatrix} 2 & -1 \end{bmatrix},$$

find

- 1. the system's Eigen values  $\lambda_{i}$ ,
- 2. the Eigen vectors  $m_i$  and modal matrix M,
- 3. the diagonalized state transition matrix  $\Phi'(t)$ ,
- 4. the state transition matrix in the original basis  $\Phi(t)$ , and
- 5. the output free response  $y_{fr}(t)$  due to an initial condition  $x(0) = [4, -1]^T$ .

## Part III

# **Modeling other systems**

### thermoflu

### Lumped-parameter modeling fluid and thermal systems

1 We now consider the lumped-parameter modeling of fluid systems and thermal systems. The linear graph-based, state-space modeling techniques of Chapters graphs to emech are called back up to service for this purpose. Recall that this method defines several types of discrete elements in an energy domain—in Chapters graphs and ss, the electrical and mechanical energy domains. Also recall from Chapter emech that energy transducing elements allow energy to flow among domains. In this chapter, we introduce fluid and thermal energy domains and discrete and transducing elements associated therewith.

2 The analogs between the mechanical and electrical systems from Chapter graphs are expanded to include fluid and thermal systems. This generalization allows us to include, in addition to electromechanical systems, inter-domain systems including electrical, mechanical, fluid, and thermal systems.

3 This chapter begins by defining discrete lumped-parameter elements for fluid and thermal systems. We then categorize these into energy source, energy storage (A-type and T-type), and energy dissapative (D-type) elements, allowing us to immediately construct linear graphs and normal trees in the manner of Chapter graphs. Then we can directly apply the methods of Chapter ss to construct state-space lumped-parameter modeling fluid systems thermal systems models of systems that include fluid and thermal elements.