thermoflu.dam State−space model of a hydroelectric dam

1 Consider the microhydroelectric dam of Example thermoflu.flutrans-1. We derived the linear graph of Fig. dam.1. In this lecture, we will derive a state-space model for the system—specifically, a state equation.

Normal tree, order, and variables

2 Now, we define a normal tree by overlaying it on the system graph in Fig. dam.1. There are six independent energy storage elements, making it a sixth-order ($n = 6$) system. We define the state vector to be

normal tree

$$x = \begin{bmatrix} P_{C_1} & P_{C_2} & Q_{L_1} & \Omega_J & i_{L_2} & v_{C_3} \end{bmatrix}^\top. \quad (1)$$

The input vector is defined as $u = \begin{bmatrix} Q_s & P_{s1} & P_{s2} \end{bmatrix}^\top$.

Elemental equations

3 Yet to be encountered is a turbine's transduction. A simple model is that the torque T_2 is proportional to the flowrate Q_1, which are both through-variables, making it a transformer, so

transformer

$$T_2 = -\alpha Q_1 \quad \text{and} \quad \Omega_2 = \frac{1}{\alpha} P_1, \quad (2)$$

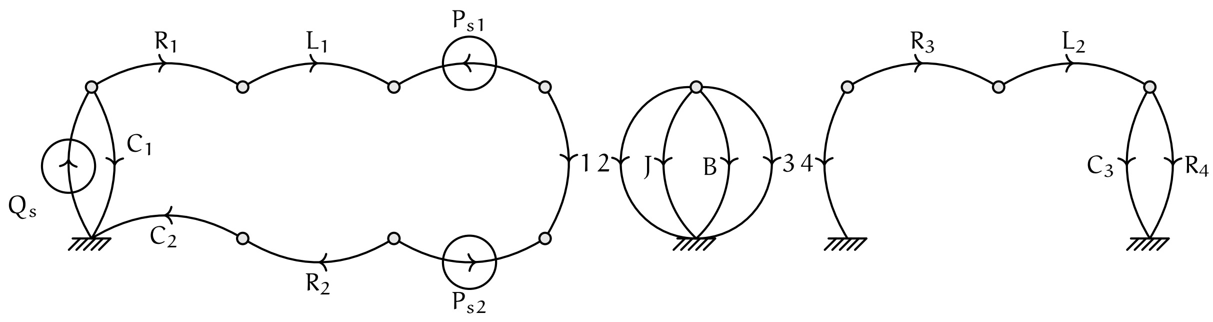

Figure dam.1: a linear graph for a microhydroelectric dam.

where α is the transformer ratio.

transformer ratio

4 The other elemental equations have been previously encountered and are listed, below.

el.	elemental eq.	el.	elemental eq.
C_1	$\frac{dP_{C_1}}{dt} = \frac{1}{C_1} Q_{C_1}$	C_3	$\frac{dv_{C_3}}{dt} = \frac{1}{C_3} i_{C_3}$
C_2	$\frac{dP_{C_2}}{dt} = \frac{1}{C_2} Q_{C_2}$	R_1	$P_{R_1} = Q_{R_1} R_1$
L_1	$\frac{dQ_{L_1}}{dt} = \frac{1}{L_1} P_{L_1}$	R_2	$P_{R_2} = Q_{R_2} R_2$
J	$\frac{d\Omega_J}{dt} = \frac{1}{J} T_J$	1	$T_2 = -\alpha Q_1$
L_2	$\frac{di_{L_2}}{dt} = \frac{1}{L_2} v_{L_2}$	2	$\Omega_2 = \frac{1}{\alpha} P_1$

el.	elemental eq.
B	$\Omega_B = \frac{1}{B} T_B$
3	$i_4 = \frac{-1}{k_m} T_3$
4	$v_4 = k_m \Omega_3$
R_3	$v_{R_3} = i_{R_3} R_3$
R_4	$i_{R_4} = \frac{1}{R_4} v_{R_4}$

Continuity and compatibility equations

5 Continuity and compatibility equations can be found in the usual way—by drawing contours and temporarily creating loops by including links in the normal tree. We proceed by drawing a table of all elements and writing a continuity equation for each branch of the normal tree and a compatibility equation for each link.

el.	eq.	el.	eq.	el.	eq.
C_1	$Q_{C_1} = Q_s - Q_{L_1}$	C_3	$i_{C_3} = i_{L_2} - i_{R_4}$	B	$\Omega_B = \Omega_J$
C_2	$Q_{C_2} = Q_{L_1}$	R_1	$Q_{R_1} = Q_{L_1}$	3	$\Omega_3 = \Omega_J$
L_1	$P_{L_1} = -P_{R_1} + P_{C_1} - P_{C_2} +$	R_2	$Q_{R_2} = Q_{L_1}$	4	$i_4 = -i_{L_2}$
	$-P_{R_2} + P_{s2} - P_1 + P_{s1}$	1	$Q_1 = Q_{L_1}$	R_3	$i_{R_3} = i_{L_2}$
J	$T_J = -T_2 - T_B - T_3$	2	$\Omega_2 = \Omega_J$	R_4	$v_{R_4} = v_{C_3}$
L_2	$v_{L_2} = -v_{R_3} + v_4 - v_{C_3}$				

State equation

6 The system of equations composed of the elemental, continuity, and compatibility equations can be reduced to the state equation. There is a substantial amount of algebra required to eliminate those variables that are neither state nor input variables. Therefore, we use the Mathematica package StateMint (Cameron N. Devine and Rico A.R. Picone. Statum. https://github.com/CameronDevine/Statum. 2018). The resulting system model is:

$$\frac{dx}{dt} = Ax + Bu,$$

$$A = \begin{bmatrix} 0 & 0 & -1/C_1 & 0 & 0 & 0 \\ 0 & 0 & 1/C_2 & 0 & 0 & 0 \\ 1/L_1 & -1/L_1 & -(R_1+R_2)/L_1 & -\alpha/L_1 & 0 & 0 \\ 0 & 0 & \alpha/J & -B/J & -k_m/J & 0 \\ 0 & 0 & 0 & k_m/L_2 & -R_3/L_2 & -1/L_2 \\ 0 & 0 & 0 & 0 & 1/C_3 & -1/(R_4 C_3) \end{bmatrix},$$

$$B = \begin{bmatrix} 1/C_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/L_1 & 1/L_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

7 The rub is estimating all these parameters.

8 The Mathematica notebook used above can be found in the source repository for this text.