freq.exe Exercises for Chapter freq

Exercise freq.gauche

Consider a system with i/o ODE

$$\ddot{y} + a\,\dot{y} + b\,y = b\,u \tag{1}$$

for constants $a, b \in \mathbb{R}$.

1. Derive the frequency response function $H(j\omega)$ and the transfer function $H(s)$. Hint: either can be found from the other.
2. Let $u(t) = 7\cos(5t + 3)$. What is the steady state forced response $y(t)$ in terms of a, b? Hint: this shouldn't require much computation.
3. Now let $u(t) = 3\,\delta(t)$, an impulse. What is the impulse response $y(t)$ in terms of the inverse Fourier transform $\mathcal{F}^{-1}$ and $H(j\omega)$? Do not substitute in for $H(j\omega)$ or inverse transform.
4. Use computer software to plot the Bode plot of $H(j\omega)$ for $a = b = 1$.
5. For $b = 1$, for what range of a will there be a complex conjugate pair of poles?[3] Hint: consider comparing the transfer function derived in part (a) to the standard form of the second-order transfer function in ??a.

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3. The following statements are equivalent. A second-order system
   - has a complex conjugate pair of poles,
   - has a complex conjugate pair of the characteristic equation,
   - has a complex conjugate pair of eigenvalues, and
   - is underdamped.

Exercise freq.tickle

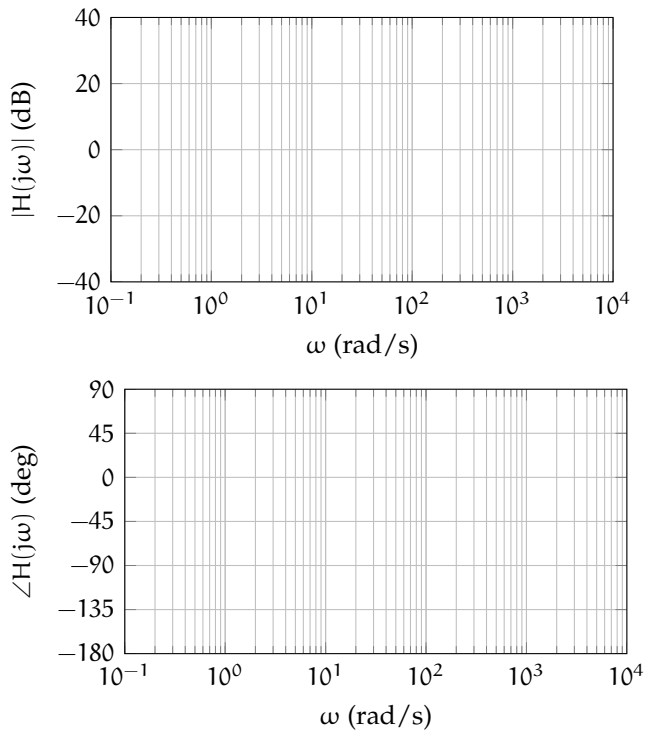
Let a transfer function H be

$$\frac{10(s + 100)}{s^2 + 2\,s + 100}. \tag{2}$$

Use H to respond to the following questions and imperatives.

a. Write H as a product of standard-form transfer functions.
b. Find the frequency response function $H(j\omega)$ without simplifying.

c. Use the axes below to sketch the Bode plot of H.



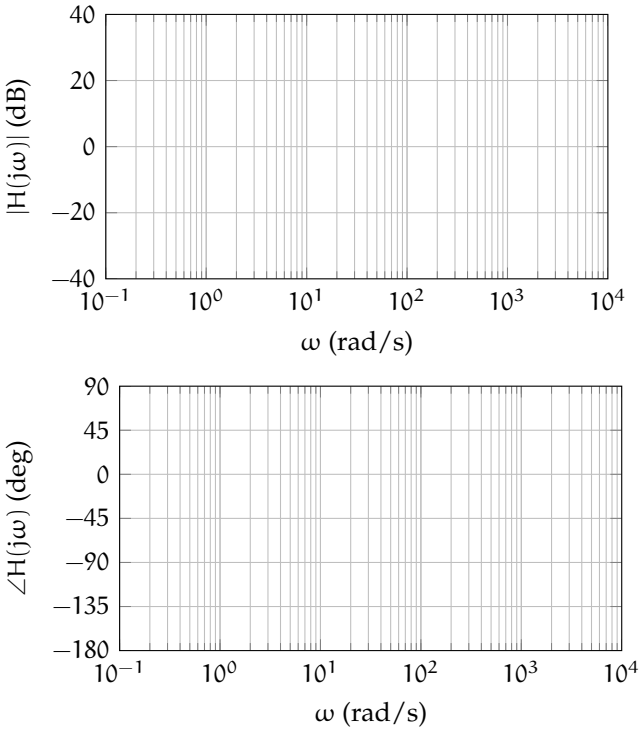
Exercise freq.me

Let a transfer function H be

$$H(s) = \frac{1000(s+10)}{(s+100)(s+1000)}.$$

Use H to respond to the following questions and imperatives.

a. Write H as a product of standard-form transfer functions.
b. Find the frequency response function $H(j\omega)$ without simplifying.
c. Use the axes below to sketch the Bode plot of H.



Exercise freq.elmo

Consider a system with transfer function

$$H(s) = \frac{100(s+9)}{(s+5)(s+6)(s^2+8s+32)}.$$

a. Identify the poles and zeros of H.
b. Derive the frequency response function $H(j\omega)$. Do not simplify the expression.
c. Create a Bode plot of H.
d. Let the system have sinusoidal input $u(t) = 2\cos(3t)$. What is the steady-state system output $y(t)$?
e. Let the system have the same sinusoidal input as previously. Simulate its forced response for nine seconds and plot it.

Part V

Laplace analysis

lap

Laplace transforms