imp.2port Impedance with two–port elements

transformers
gyrators

1 The two types of energy transducing elements, transformers and gyrators, "reflect" or "transmit" impedance through themselves, such that they are "felt" on the other side.

transformer

2 For a transformer, the elemental equations are

$$\mathcal{V}_2(t) = \mathcal{V}_1(t)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(t) = -\mathrm{TF}\mathcal{F}_1(t), \tag{1}$$

the Laplace transforms of which are

$$\mathcal{V}_2(s) = \mathcal{V}_1(s)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(s) = -\mathrm{TF}\mathcal{F}_1(s). \tag{2}$$

3 If, on the 2-side, the input impedance is Z_3, as in Fig. 2port.1, the equations of Eq. 2 are subject to the continuity and compatibility equations

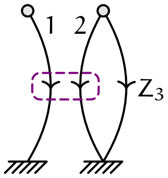

Figure 2port.1:

$$\mathcal{V}_2 = \mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_2 = -\mathcal{F}_3. \tag{3}$$

Substituting these into Eq. 2 and solving for $\mathcal{V}_1$ and $\mathcal{F}_1$,

$$\mathcal{V}_1 = \mathrm{TF}\mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_1 = \mathcal{F}_3/\mathrm{TF}. \tag{4}$$

The elemental equation for element 3 is $\mathcal{V}_3 = \mathcal{F}_3 Z_3$, which can be substituted into the through-variable equation to yield

4 Working our way back from $\mathcal{V}_3$ to $\mathcal{V}_1$, we apply the compatibility equation $\mathcal{V}_2 = \mathcal{V}_3$ and the elemental equation $\mathcal{V}_2 = \mathcal{V}_1/\mathrm{TF}$, as follows:

effective input impedance

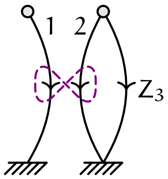

Figure 2port.2:

Solving for the effective input impedance Z_1,

gyrator

$$Z_1 \equiv \frac{\mathcal{V}_1(s)}{\mathcal{F}_1(s)} \tag{5}$$

$$= TF^2 Z_3. \tag{6}$$

5 For a gyrator with gyrator modulus GY, in the configuration shown in Fig. 2port.2, a similar derivation yields the effective input impedance Z_1,

effective input impedance

$$Z_1 = GY^2/Z_3. \tag{7}$$

Example imp.2port−1

re: input impedance of fluid system with transducer

Draw a linear graph of the fluid system. What is the input impedance for an input force to the piston?