math.matrix Matrix inverses

This is a guide to inverting $1\times 1, 2\times 2,$ and $n\times n$ matrices.

Let A be the 1×1 matrix

$$\mathsf{A}=\left[\mathfrak{a}\right].$$

The inverse is simply the reciprocal:

$$A^{-1} = \left[1/\alpha\right].$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}$$
$$= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}.$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C,$$

where adj is the adjoint of C.

adjoint