fun.vir Voltage, current, resistance, and all that

Two quantities will be of special importance in analyzing and designing electronic systems: voltage and current. The relationship between them defines a third important quantity: resistance (more generally, impedance). Momentarily, we will define each of these, but we start with the fundamental quantity in electronics.

voltage
current
resistance

Definition fun.1: electric charge

Electric charge (or simply charge) is a property of matter that describes the attractive or repulsive force acting on the matter in an electric field. At the microscopic level, charge is quantized into charges of subatomic particles such as protons and electrons, which have opposite charges e and $-e$, where e is the elementary charge.

Charge has derived SI unit coulomb with symbol C. It is considered to be a conserved quantity.

coulomb
conserved quantity

Voltage

Definition fun.2: voltage

Voltage is the difference in electrical potential energy of a unit of charge moved between two locations in an electrical field.

Voltage is typically given the variable v and has derived SI unit volt with symbol V.
Voltage is always defined by referring to two locations. Sometimes one of these locations is implicitly ground—an arbitrarily-defined reference (datum) voltage considered to have zero electrical potential energy—such that we can talk about the voltage “at this” or “at that” location by implicit reference to ground. It is

volt
ground

good form to describe the voltage as being "between" two locations or "across" an element.

Current

Definition fun.3: current

Current is a flow of charge.

Current is typically denoted i and has derived SI unit ampere with symbol A.

ampere

We typically generate voltage by doing work on charges. Conversely, we get currents by placing voltage across matter through which current can flow. This implies that voltage causes current. Causality here is quite complex, but I will posit the following proposition. We typically observe current when applying voltage, so from a phenomenological point-of-view, it is natural to consider voltage causal of current.[2]

2. Note that subtlety emerges not only when considering fields, small distances, and short durations—it also emerges when we consider certain circuit elements that are exhibit behavior related to the time rate of change of voltage or current.

Circuits

Electric circuits are dynamic electrical systems in which charge accumulates in and flows through elements. Circuit elements are connected via metallic conductors called wires, which ideally have the same voltage (relative to, say, ground) everywhere.

circuit

wire

Circuit topology

A circuit has a few basic topological features.

A circuit node is a continuous region of a circuit that has the same voltage everywhere. A node is an idealized concept that is approximate in most instantiations.

node

A circuit element is a region of a circuit considered to have properties distinct from the surrounding circuit. Examples of elements are resistors, capacitors, inductors, and sources.

element

A circuit element has terminals through which it connects to a circuit.

terminals

Circuit elements in parallel are those that have

parallel

two terminals, each of which is shared by another element's two terminals.

series

Circuit elements in series are those that have two terminals, only one of which is shared between them and this one cannot be shared with any other element.

Element types

The following are common types of circuit element.

energy storage element

energy dissipative element

energy source element

energy transducing element

- Energy storage elements store energy in electric (capacitors) or magnetic (inductors) fields.
- Energy dissipative elements dissipate energy from a circuit, typically as heat, such as in a resistor.
- Energy source elements provide external energy to the circuit (e.g. batteries).
- Energy transducing elements convert electronic energy to another form (e.g. motors convert electric to mechanical energy.)

Power

Power is the time rate of change of energy. Let us now define electric power.

Definition fun.4: power

The instantaneous electric power $\mathcal{P}$ into a circuit element is defined as the product of the voltage v across and the current i through it at a given time t:

$$\mathcal{P}(t) = v(t)i(t). \tag{1}$$

Power typically goes into:

- heat (usually),
- mechanical work (motors),
- radiated energy (lamps, transmitters), or
- stored energy (batteries, capacitors).

Box fun.1 terminological note

"[D]on't call current 'amperage'; that's strictly bush-league. The same caution will apply to the term 'ohmage' ...."
—Horowitz & Hill, The Art of Electronics

Kirchhoff's laws

Gustav Kirchhoff formulated two laws fundamental to circuit analysis.
Kirchhoff's current law (KCL) depends on the fact that charge is a conserved quantity. Therefore, the charge flowing in a node is equal to that flowing out, which implies KCL.

Definition fun.5: Kirchhoff's current law
The current in a node is equal to the current out.

KCL implies that the sum of the current into a node must be zero. Assume, for instance, that k wires with currents i_j connect to form a node. Kirchhoff's current law states that

$$\sum_{j=1}^{k} i_j = 0. \tag{2}$$

It can be discovered empirically that elements connected in parallel have the same voltage across them. This doesn't mean they share the same current, but it does imply Kirchhoff's voltage law (KVL).

Definition fun.6: Kirchhoff's voltage law
The sum of the voltage drops around any closed loop is zero.[a]

a. A loop is a series of elements that begins and ends at the same node.

KVL implies that the voltage drops across elements that form a loop must be zero. Assume, for instance, that k elements with voltage drops v_j form a loop. KVL states that

$$\sum_{j=1}^{k} v_j = 0. \tag{3}$$

Ohm's law

Much of electronics is about the relationship between a voltage and a corresponding current. Applying a voltage to a material typically induces a current through it. The functional relationship between v and i is of the utmost importance to the analysis and design of circuits.

The simplest relationship is known as Ohm's law, for which we will first need the concept of resistance.

Definition fun.7: resistance

Let a circuit element have voltage v and current i. The resistance R is defined as the ratio

$$R = v/i \tag{4}$$

Now we are ready to define Ohm's law.

Definition fun.8: Ohm's law

Some materials such as conductors in certain environments exhibit approximately constant resistance.

This is pretty weak. However, it's still quite useful, as we'll see. With it we can assume, for certain elements and situations, that the resistance of the element is a static property and that the voltage and current are proportional. We call such elements resistors.

resistor

Combining resistance

Resistors can be connected together in different topologies to form composite elements that exhibit "equivalent" resistances of their own.

K resistors with resistances R_j connected in series have equivalent resistance R_e given by the expression

$$R_e = \sum_{j=1}^{K} R_j. \tag{5}$$

K resistors with resistances R_j connected in parallel have equivalent resistance R_e given by the expression

$$R_e = 1/\sum_{j=1}^{K} 1/R_j. \tag{6}$$

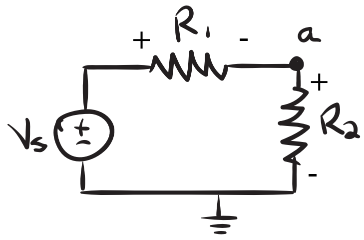
In the special case of two resistors with resistances R_1 and R_2,

(7)

Example fun.vir-1 **re: understanding a circuit**

Answer the questions below about the circuit shown. Voltage across and current through a circuit element x are denoted v_x and i_x. Signs are defined on the diagram.

1. What does it mean if we refer to the voltage at node a?
2. What is the current i_{R_2} through R_2 at a given time t in terms of the power it is dissipating $\mathcal{P}_{R_2}$ and the voltage across it v_{R_2}?
3. If $V_s(t) = 5$ V and $v_{R_1} = 3$ V, what is v_{R_2}?
4. What is the equivalent resistance of the resistors R_1 and R_2 combined as in the circuit?
5. If $v_{R_1} = 3$ V and $R_1 = 100\ \Omega$, what is i_{R_2}?


R1
+
-
a
+
R2
-
Vs