ssan.imp Impedance

With complex representations for voltage and current, we can introduce the concept of impedance.

impedance

Definition ssan.1: impedance

Impedance Z is the complex ratio of voltage v to current i of a circuit element:

$$Z = \frac{v}{i}.$$

The real part $\text{Re}(Z)$ is called the resistance and the imaginary part $\text{Im}(Z)$ is called the reactance. As with complex voltage and current, we can represent the impedance as a phasor.

resistance

reactance

Note that Definition ssan.1 is a generalization of Ohm's law. In fact, we call the following expression generalized Ohm's law:

generalized Ohm's law

$$v = iZ. \tag{1}$$

Impedance of circuit elements

The impedance of each of the three passive circuit elements we've considered thus far are listed, below. Wherever it appears, ω is the angular frequency of the element's voltage and current.

resistor For a resistor with resistance R, the impedance is all real:

capacitor For a capacitor with capacitance C, the impedance is all imaginary:

inductor For an inductor with inductance L, the impedance is all imaginary:

These are represented in the complex plane in Fig. imp.1.

Combining the impedance of multiple elements

As with resistance, the impedance of multiple elements may be combined to find an effective impedance.

effective impedance

K elements with impedances Z_j connected in series have equivalent impedance Z_e given by the expression

$$Z_e = \sum_{j=1}^{K} Z_j. \tag{2}$$

K elements with impedances Z_j connected in parallel have equivalent impedance Z_e given by the expression

$$Z_e = 1/\sum_{j=1}^{K} 1/Z_j. \tag{3}$$

In the special case of two elements with impedances Z_1 and Z_2,

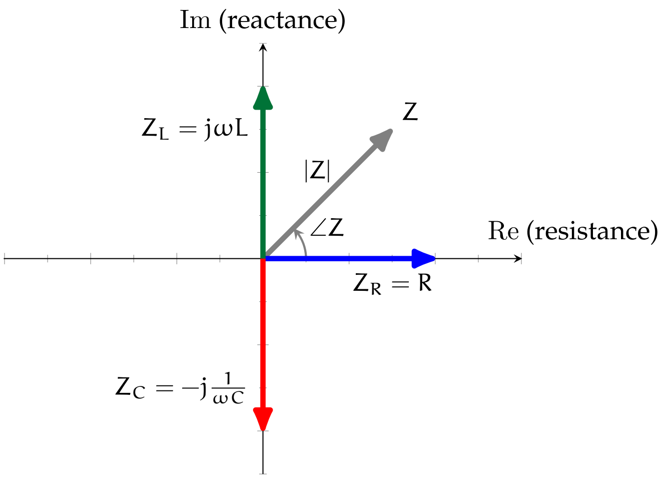

Figure imp.1: the impedance of a resistor Z_R, a capacitor Z_C, and an inductor Z_L in the complex plane.

(4)

Example ssan.imp−1 **re: combining impedance and phasors**

Given the circuit shown with voltage source $V_s(t) = Ae^{j\phi}$, what is the total impedance at the source?.