# nlnmul.dio Diodes

Diodes are single-port nonlinear elements that, approximately, conduct current in only one direction. We will consider the ubiquitous semiconductor diode, varieties of which include the light-emitting diode (LED), photodiode (for light sensing), Schottky diode (for fast switching), and Zener diode (for voltage regulation). See Fig. dio.1 for corresponding circuit symbols.

**semiconductor diode**
**light−emitting diode (LED)**
**photodiode**
**Schottky diode**
**Zener diode**

In most cases, we use the diode to conduct current in one direction and block reverse current.[4] When conducting current in its forward direction, it is said to have forward-bias; when blocking current flow in its reverse direction, it is said to have reverse-bias. If the reverse breakdown voltage is reached, current will flow in the reverse direction. It is important to check that a circuit design does not subject a diode to its breakdown voltage, except in special cases (e.g. when using a Zener diode).

4. The paradigmatic exception is the Zener diode, which is typically used in reverse bias in order to take advantage of its highly stable reverse bias voltage over a large range of reverse current. We will not consider this application here.

**forward−bias**
**reverse−bias**
**breakdown voltage**

We begin with a nonlinear model of the voltage-current $v_D$-$i_D$ relationship. Let

- $I_s$ be the saturation current (typically ~$10^{-12}$ A) and
- $V_{TH} = k_b T/e$ be the thermal voltage (at room temperature ~25 mV) with[5]
  - $k_b$ the Boltzmann constant,
  - $e$ the fundamental charge, and
  - $T$ the diode temperature.

5. Unless otherwise specified, it is usually reasonable to assume room-temperature operation.

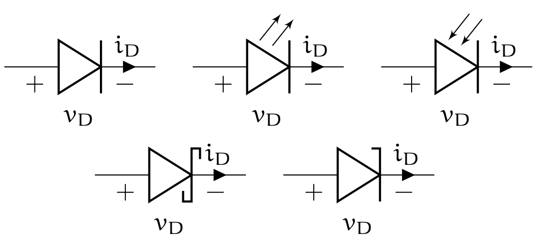



**Figure dio.1:** diode symbols. From left to right, the generic symbol, LED, photodiode, Schottky, Zener.

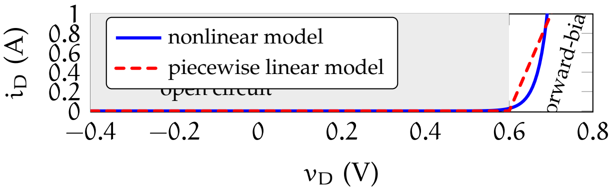



**Figure dio.2:** the voltage-current relationship in the nonlinear and piecewise linear models. In the figure, $R_d = 0.1\ \Omega$.

**Equation 1 nonlinear diode model**

See Fig. dio.2 for a plot of this function. One can analyze circuits with diodes using the methods of Chapter can and Eq. 1 as the diode's elemental equation. A nonlinear set of equations results, which typically require numerical solution techniques.

### A piecewise linear model

**ideal diode**

An ideal diode is one that is a perfect insulator (open circuit, $i_D = 0$) for $v_D < 0$ conductor for $v_D > 0$. We use the symbol shown in Fig. dio.3 for the ideal diode. At times, the ideal diode is sufficient to model a diode; often, however, we prefer a more accurate model that is piecewise linear.

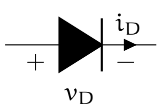



**Figure dio.3:** circuit symbol for an ideal diode. Note that this is a nonstandard use of this symbol.

**piecewise linear model**

The piecewise linear model is shown in Fig. dio.4. It includes an ideal diode in series with a fixed voltage drop of 0.6 V and a resistor with resistance $R_d$. This approximates the nonlinear model with two linear approximations.

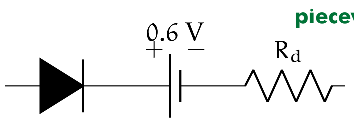



**Figure dio.4:** piecewise linear model.

**Equation 2 piecewise linear diode model**

See Fig. dio.2 for a plot of this function and a comparison to the nonlinear model.
The slope in forward-bias is $1/R_d$. This model's effectiveness is highly dependant on $R_d$, so an operating point must be chosen and $R_d$ chosen to match most closely with the nonlinear model near that operating point.

**operating point**

### Method of assumed states

**method of assumed states**

The method of assumed states is a method for using linear circuit analysis to analyze circuits with nonlinear components. The method is summarized in the following steps.

1. Begin at the initial time $t = 0$.
2. Replace each diode in the circuit diagram with the piecewise linear diode model.
3. Proceed with the circuit analysis of Chapter can, ignoring the elemental equations for the ideal diodes $D_i$. Your system of equations will have unknown ideal diode current $i_{D_i}$ and voltage $v_{D_i}$. Simplify it to the extent possible.
4. Guess the current state of each ideal diode: `ON` or `OFF`. For each ideal diode $D_i$ guessed to be `ON`,

$$\text{set } v_{D_i} = 0 \quad \text{and assume that} \quad i_{D_i} > 0. \tag{3}$$

For each ideal diode assumed to be `OFF`,

$$\text{set } i_{D_i} = 0 \quad \text{and assume that} \quad v_{D_i} < 0. \tag{4}$$

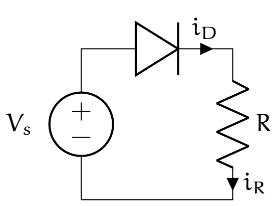
For $n$ diodes in the circuit, there are $2^n$ possibilities at each moment in time. Guess just one to start.

5. If even one diode violates its assumption from above, dismiss the results and return to step 4 and choose a different combination of assumed states (consider flipping the assumptions on those diodes that violated the old assumptions).
6. If not even one diode violates its assumptions, this is the correct solution for this moment in time.
7. This solution is valid for as long as its assumptions are valid. Once they fail, go back to step 4.

Since impedance methods are valid only for linear circuits, steady-state analyses should proceed with the same process outlined above. With a periodic input, a periodic (steady) solution may emerge.

**Example nlnmul.dio−1** **re: half wave rectifier**

Given the circuit shown with voltage source $V_s(t) = 3\cos 2\pi t$, what is the output $v_R$? Explain why this might be called a "half-wave rectifier." Let $R = 10\ \Omega$.



## An algorithm for determining $R_d$

The piecewise linear approximation of the exponential diode current will never be great, but we can at least try to choose $R_d$ in a somewhat optimal way, recognizing that when highly accurate results are required, there's no substitute for the nonlinear model.

Consider the algorithm of Fig. dio.6. Initially set

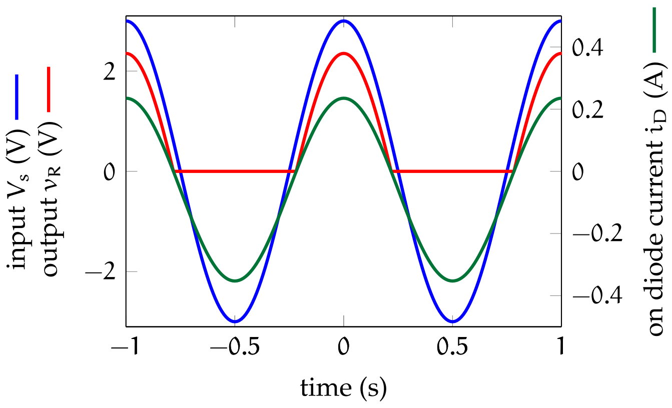



**Figure dio.5:** the input and output voltage of the half-wave rectifier circuit of Example nlnmul.dio-1. Note that the "on" diode subcircuit is valid for $i_D > 0$ and the "off" diode circuit is valid for $i_D < 0$.

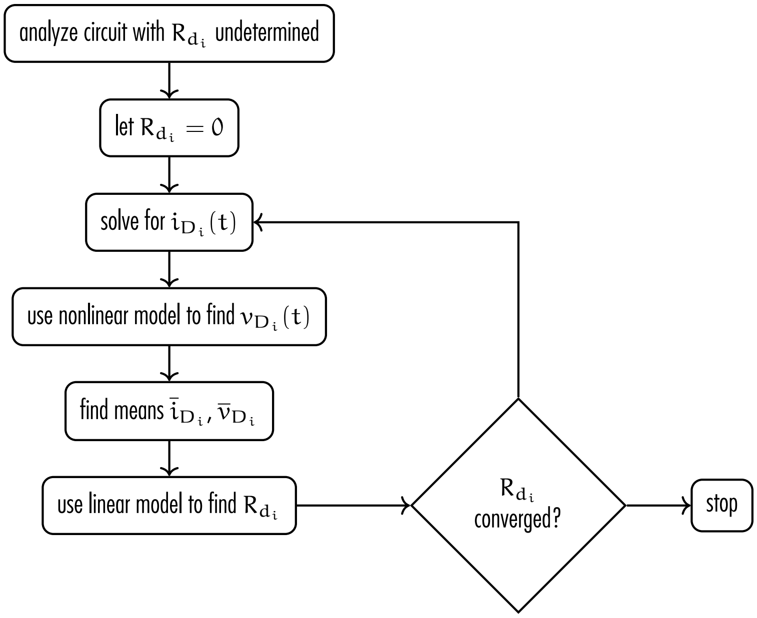



**Figure dio.6:** an algorithm for determining $R_{d_i}$.

to zero the diode resistances $R_{d_i}$ of each resistor. Solve for each diode current $i_{D_i}(t)$, then use this to find $v_{D_i}(t)$ from the nonlinear model of Eq. 1:

$$v_{D_i}(t) = V_{TH} \ln(i_{D_i}(t)/I_s + 1). \tag{5}$$

Now take the means of these signals (assuming steady state oscillation) over a period $T$, excluding the time $T_0$ during which the diode voltage was in reverse-bias:[6]

$$\bar{i}_{D_i} = \frac{1}{T - T_0} \int_{t_0}^{t_0+T} i_{D_i}(\tau) d\tau \tag{6a}$$

$$\bar{v}_{D_i} = \frac{1}{T - T_0} \int_{t_0}^{t_0+T} v_{D_i}(\tau) d\tau. \tag{6b}$$

Now us the piecewise linear model of ?? to estimate $R_{d_i}$:

$$R_{d_i} = \frac{\bar{v}_{D_i} - 0.6V}{\bar{i}_{D_i}}. \tag{7}$$

We can use this estimate of $R_{d_i}$ to re-analyze the circuit and repeat the same process of deriving a new estimate of $R_{d_i}$. This process should converge on an estimate of $R_{d_i}$ that is in some sense optimal.

6. Note that if $T_0$ is ignored, our estimate of $R_d$ will include the effects of time during which no current is flowing and the diode is in reverse-bias, during which time $R_d$ is not applicable.

Note that if, during this iterative process, one finds $\overline{v}_{D_i} < 0.6$ V, a negative $R_d$ will result. At this point, a couple different reasonable approaches can be taken:

1. just use $R_{d_i} = 0$ or
2. use some reasonably central value of $\overline{v}_{D_i} > 0.6$ V.

The second case is preferred if $v_{D_i}(t)$ spends much time above $0.6$ V. But usually, if it spends much time, the mean $\overline{v}_{D_i}$ should be great enough to avoid this situation. Circuits that tend to express this behavior are those with high impedance and correspondingly low currents.