sets.logic Logical connectives and quantifiers

In order to make compound propositions, we need to define logical connectives. In order to specify quantities of variables, we need to define logical quantifiers. The following is a form of first-order logic (Wikipedia, First-order logic — Wikipedia, The Free Encyclopedia).

first–order logic

Logical connectives

A proposition can be either true $\top$ and false $\bot$. When it does not contain a logical connective, it is called an atomistic proposition. To combine propositions into a compound proposition, we require logical connectives. They are not ($\neg$), and ($\wedge$), and or ($\vee$). Table logic.1 is a truth table for a number of connectives.

atomistic proposition
compound proposition
logical connectives
not
and
or
truth table

Quantifiers

Logical quantifiers allow us to indicate the quantity of a variable. The universal quantifier symbol $\forall$ means "for all". For instance, let A be a set; then $\forall a \in A$ means "for all elements in A" and gives this quantity variable a. The existential quantifier $\exists$ means "there exists at least one" or "for some". For instance, let A be a set; then $\exists a \in A \ldots$ means "there exists at least one element a in A ...."

universal quantifier symbol
existential quantifier

Table logic.1: a truth table for logical connectives. The first two columns are the truth values of propositions p and q; the rest are outputs.

		not	and	or	nand	nor	xor	xnor
p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \uparrow q$	$p \downarrow q$	$p \veebar q$	$p \Leftrightarrow q$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\top$