prob.prob Basic probability theory

The mathematical model for a class of measurements is called the probability space and is composed of a mathematical triple of a sample space Ω, σ-algebra $\mathcal{F}$, and probability measure P, typically denoted $(\Omega, \mathcal{F}, P)$, each of which we will consider in turn (Wikipedia. Probability space — Wikipedia, The Free Encyclopedia. http://en.wikipedia.org/w/index.php?title=Probability%20space&oldid=914939789. [Online; accessed 31-October-2019]. 2019).

probability space

The sample space Ω of an experiment is the set representing all possible outcomes of the experiment. If a coin is flipped, the sample space is $\Omega = \{H, T\}$, where H is heads and T is tails. If a coin is flipped twice, the sample space could be

sample space

outcomes

However, the same experiment can have different sample spaces. For instance, for two coin flips, we could also choose

We base our choice of Ω on the problem at hand.

An event is a subset of the sample space. That is, an event corresponds to a yes-or-no question about the experiment. For instance, event A (remember: $A \subseteq \Omega$) in the coin flipping experiment (two flips) might be $A = \{HT, TH\}$. A is an event that corresponds to the question, "Is the second flip different than the first?" A is the event for which the answer is "yes."

event

Algebra of events

Because events are sets, we can perform the usual set operations with them.

Example prob.prob−1 **re: set operations with events**

Consider a toss of a single die. We choose the sample space to be $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let the following define events.

$$A \equiv \{\text{the result is even}\} = \{2, 4, 6\}$$
$$B \equiv \{\text{the result is greater than 2}\} = \{3, 4, 5, 6\}.$$

Find the following event combinations:

$$A \cup B \quad A \cap B \quad A \setminus B \quad B \setminus A \quad \overline{A} \setminus B.$$

σ−algebra

The σ-algebra $\mathcal{F}$ is the collection of events of interest. Often, $\mathcal{F}$ is the set of all possible events given a sample space Ω, which is just the power set of Ω (Wikipedia, Probability space — Wikipedia, The Free Encyclopedia). When referring to an event, we often state that it is an element of $\mathcal{F}$. For instance, we might say an event $A \in \mathcal{F}$.

We're finally ready to assign probabilities to events. We define the probability measure $P : \mathcal{F} \to [0, 1]$ to be a function satisfying the following conditions.

probability measure

1. For every event $A \in \mathcal{F}$, the probability measure of A is greater than or equal to zero—i.e. $P(A) \geqslant 0$.
2. If an event is the entire sample space, its probability measure is unity—i.e. if $A = \Omega$, $P(A) = 1$.

3. If events $A_1, A_2, \cdots$ are disjoint sets (no elements in common), then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

We conclude the basics by observing four facts that can be proven from the definitions above.

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