prob.rando Random variables

Probabilities are useful even when they do not deal strictly with events. It often occurs that we measure something that has randomness associated with it. We use random variables to represent these measurements.

random variable

A random variable $X : \Omega \to \mathbb{R}$ is a function that maps an outcome ω from the sample space Ω to a real number $x \in \mathbb{R}$, as shown in Fig. rando.1. A random variable will be denoted with a capital letter (e.g. X and K) and a specific value that it maps to (the value) will be denoted with a lowercase letter (e.g. x and k).

discrete random variable

continuous random variable

A discrete random variable K is one that takes on discrete values. A continuous random variable X is one that takes on continuous values.

Example prob.rando−1

re: dice again

Roll two unbiased dice. Let K be a random variable representing the sum of the two. Let $P(k)$ be the probability of the result $k \in K$. Plot and interpret $P(k)$.

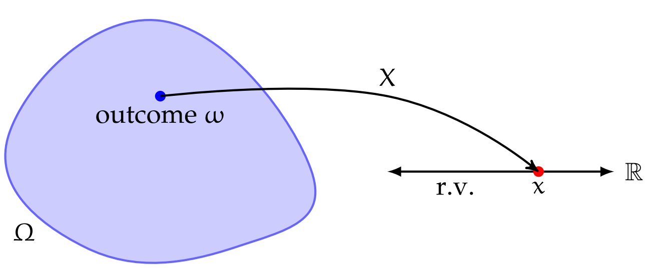

Figure rando.1: a random variable X maps an outcome $\omega \in \Omega$ to an $x \in \mathbb{R}$.

Example prob.rando–2 re: Johnson–Nyquist noise

A resistor at nonzero temperature without any applied voltage exhibits an interesting phenomenon: its voltage randomly fluctuates. This is called Johnson-Nyquist noise and is a result of thermal excitation of charge carriers (electrons, typically). For a given resistor and measurement system, let the probability density function f_V of the voltage V across an unrealistically hot resistor be

$$f_V(V) = \frac{1}{\sqrt{\pi}} e^{-V^2}.$$

Plot and interpret the meaning of this function.