prob.E Expectation

Recall that a random variable is a function $X : \Omega \to \mathbb{R}$ that maps from the sample space to the reals. Random variables are the arguments of probability mass functions (PMFs) and probability density functions (PDFs).

expected value
expectation

The expected value (or expectation) of a random variable is akin to its "average value" and depends on its PMF or PDF. The expected value of a random variable X is denoted $\langle X \rangle$ or $E[X]$. There are two definitions of the expectation, one for a discrete random variable, the other for a continuous random variable. Before we define, them, however, it is useful to predefine the most fundamental property of a random variable, its mean.

mean

Definition prob.1: mean

The mean of a random variable X is defined as

$$m_X = E[X].$$

Let's begin with a discrete random variable.

Definition prob.2: expectation of a discrete random variable

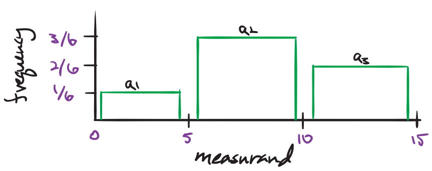
Let K be a discrete random variable and f its PMF. The expected value of K is defined as

$$E[K] = \sum_{\forall k} k f(k).$$

Example prob.E−1

re: expectation of a discrete random variable

Given a discrete random variable K with PMF shown below, what is its mean m_K?



Let us now turn to the expectation of a continuous random variable.

Definition prob.3: expectation of a continuous random variable

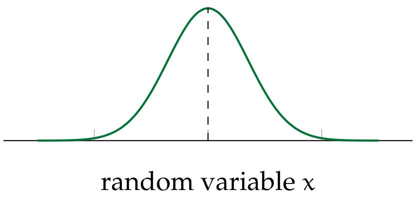
Let X be a continuous random variable and f its PDF. The expected value of X is defined as

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx.$$

Example prob.E−2

Given a continuous random variable X with Gaussian PDF f, what is the expected value of X?

re: expectation of a continuous random variable



Due to its sum or integral form, the expected value $\mathrm{E}[\cdot]$ has some familiar properties for random variables X and Y and reals a and b.

$$\mathrm{E}[a] = a \tag{1a}$$
$$\mathrm{E}[X + a] = \mathrm{E}[X] + a \tag{1b}$$
$$\mathrm{E}[aX] = a\,\mathrm{E}[X] \tag{1c}$$
$$\mathrm{E}[\mathrm{E}[X]] = \mathrm{E}[X] \tag{1d}$$
$$\mathrm{E}[aX + bY] = a\,\mathrm{E}[X] + b\,\mathrm{E}[Y]\,. \tag{1e}$$