prob.moments Central moments

Given a probability mass function (PMF) or probability density function (PDF) of a random variable, several useful parameters of the random variable can be computed. These are called central moments, which quantify parameters relative to its mean.

central moments

Definition prob.4: central moments

The nth central moment of random variable X, with PDF f, is defined as

$$E\left[(X-\mu_X)^n\right] = \int_{-\infty}^{\infty} (x-\mu_X)^n f(x)dx.$$

For discrete random variable K with PMF f,

$$E\left[(K-\mu_K)^n\right] = \sum_{\forall k}^{\infty} (k-\mu_K)^n f(k).$$

Example prob.moments−1 **re: first moment**

Prove that the first moment of continuous random variable X is zero.

The second central moment of random variable X is called the variance and is denoted

variance

$$\sigma_X^2 \quad \text{or} \quad \mathrm{Var}[X] \quad \text{or} \quad E\left[(X-\mu_X)^2\right]. \tag{1}$$

The variance is a measure of the width or spread of the PMF or PDF. We usually compute the variance with the formula

Other properties of variance include, for real constant c,

$$\operatorname{Var}[c] = 0 \tag{2}$$

$$\operatorname{Var}[X + c] = \operatorname{Var}[X] \tag{3}$$

$$\operatorname{Var}[cX] = c^2 \operatorname{Var}[X]\,. \tag{4}$$

The standard deviation is defined as

standard deviation

Although the variance is mathematically more convenient, the standard deviation has the same physical units as X, so it is often the more physically meaningful quantity. Due to its meaning as the width or spread of the probability distribution, and its sharing of physical units, it is a convenient choice for error bars on plots of a random variable.

The skewness $\operatorname{Skew}[X]$ is a normalized third central moment:

skewness

$$\operatorname{Skew}[X] = \frac{E\left[(X - \mu_X)^3\right]}{\sigma_X^3}. \tag{5}$$

Skewness is a measure of asymmetry of a random variable's PDF or PMF. For a symmetric PMF or PDF, such as the Gaussian PDF, $\operatorname{Skew}[X] = 0$.

asymmetry

The kurtosis $\operatorname{Kurt}[X]$ is a normalized fourth central moment:

kurtosis

$$\operatorname{Kurt}[X] = \frac{E\left[(X - \mu_X)^4\right]}{\sigma_X^4}. \tag{6}$$

Kurtosis is a measure of the tailedness of a random variable's PDF or PMF. "Heavier" tails yield higher kurtosis.

tailedness

A Gaussian random variable has PDF with kurtosis 3. Given that for Gaussians both skewness and kurtosis have nice values (0 and 3), we can think of skewness and and kurtosis as measures of similarity to the Gaussian PDF.