stats.exe Exercises for Chapter stats

Exercise stats.brew

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You need to know the duration of time a certain stage of a brewing process takes. You set up an automated test environment that repeats the test 100 times, recorded in the following JSON[4] data file:

http://ricopic.one/mathematical_foundations/source/brew.json

Perform the following analysis.

a. Download and parse the JSON file (it contains a single array).
b. Estimate the duration of the process from the sample.
c. Choose and justify an assumed probability density function for the random variable duration.
d. Use this PDF model to compute a 99 percent confidence interval for your duration estimate.
e. Compute your duration confidence interval for the range of confidence values $[85, 99.99]$ percent.[5]
f. Plot the confidence intervals over the range of confidence in said intervals.

4. JSON is a simple and common programming language-independent data format. For parsing it with Matlab, see `jsondecode` here: mathworks.com/help/matlab/ref/jsondecode.html. For parsing it with Python, see the module `json` here: docs.python.org/library/json.

5. Consider using a z- or t-score inverse CDF lookup function like `t.ppf` from `scipy.stats`.

Exercise stats.laboritorium

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Use linear regression techniques to find the values of a, b, c, and d, in a cubic function of the form,

$$f(x) = ax^3 + bx^2 + cx + d,$$

using the data below.

x	$f(x)$
-2.0	-4.7
-1.5	-1.9
-1.0	1.5
-0.5	1.5
0.0	1.4
0.6	0.3
1.1	-1.5
1.6	0.0
2.1	0.6
2.6	4.2

Exercise stats.robotization

Use linear regression techniques to find the value of τ in the function,

$$f(t) = 1 - e^{\frac{-t^2}{\tau}}$$

Using the data below.

t	$f(t)$
0.1	0.02
0.6	0.34
1.1	0.74
1.6	0.94
2.1	0.98

Vector calculus

A great many physical situations of interest to engineers can be described by calculus. It can describe how quantities continuously change over (say) time and gives tools for computing other quantities. We assume familiarity with the fundamentals of calculus: limit, series, derivative, and integral. From these and a basic grasp of vectors, we will outline some of the highlights of vector calculus. Vector calculus is particularly useful for describing the physics of, for instance, the following.

calculus

limit
series
derivative
integral

vector calculus

mechanics of particles wherein is studied the motion of particles and the forcing causes thereof

rigid−body mechanics wherein is studied the motion, rotational and translational, and its forcing causes, of bodies considered rigid (undeformable)

solid mechanics wherein is studied the motion and deformation, and their forcing causes, of continuous solid bodies (those that retain a specific resting shape)

fluid mechanics wherein is studied the motion and its forcing causes of fluids (liquids, gases, plasmas)

heat transfer wherein is studied the movement of thermal energy through and among bodies

electromagnetism wherein is studied the motion and its forcing causes of electrically charged particles

This last example was in fact very influential in the original development of both vector calculus and complex analysis.[1] It is not an exaggeration to say that the topics above comprise the majority of physical topics of interest in engineering.

complex analysis

1. For an introduction to complex analysis, see Kreyszig. (Kreyszig, Advanced Engineering Mathematics, Part D)

A good introduction to vector calculus is given by Kreyszig.[2] Perhaps the most famous and enjoyable treatment is given by Schey[3] in the adorably titled Div, Grad, Curl and All that.

2. ibidem, Chapters 9, 10.

3. H.M. Schey. Div, Grad, Curl, and All that: An Informal Text on Vector Calculus. W.W. Norton, 2005. ISBN: 9780393925166.

It is important to note that in much of what follows, we will describe (typically the three-dimensional space of our lived experience) as a euclidean vector space: an n-dimensional vector space isomorphic to $\mathbb{R}^n$. As we know from linear algebra, any vector $\boldsymbol{v} \in \mathbb{R}^n$ can be expressed in any number of bases. That is, the vector $\boldsymbol{v}$ is a basis-free object with multiple basis representations. The components and basis vectors of a vector change with basis changes, but the vector itself is invariant. A coordinate system is in fact just a basis. We are most familiar, of course, with Cartesian coordinates, which is the specific orthonormal basis $\mathbf{b}$ for $\mathbb{R}^n$:

euclidean vector space

bases

components

basis vectors

invariant

coordinate system

Cartesian coordinates

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \quad \cdots, \quad \mathbf{b}_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \qquad (1)$$

Manifolds are spaces that appear locally as $\mathbb{R}^n$, but can be globally rather different and can describe non-euclidean geometry wherein euclidean geometry's parallel postulate is invalid. Calculus on manifolds is the focus of differential geometry, a subset of which we can consider our current study. A motivation for further study of differential geometry is that it is very convenient when dealing with advanced applications of mechanics, such as rigid-body mechanics of robots and vehicles. A very nice mathematical introduction is given by Lee[4] and

manifolds

non−euclidean geometry

parallel postulate

differential geometry

4. John M. Lee. Introduction to Smooth Manifolds. second. volume 218. Graduate Texts in Mathematics. Springer, 2012.

Bullo and Lewis[5] give a compact presentation in the context of robotics.

Vector fields have several important properties of interest we'll explore in this chapter. Our goal is to gain an intuition of these properties and be able to perform basic calculation.

5. Francesco Bullo and Andrew D. Lewis. Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems. byeditorJ.E. Marsden, L. Sirovich and M. Golubitsky. Springer, 2005.