C.03 Matrix inverses

This is a guide to inverting 1×1, 2×2, and $n \times n$ matrices.

Let A be the 1×1 matrix

$$A = \begin{bmatrix} a \end{bmatrix}.$$

The inverse is simply the reciprocal:

$$A^{-1} = \begin{bmatrix} 1/a \end{bmatrix}.$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$\begin{aligned} B^{-1} &= \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \\ &= \frac{1}{b_{11} b_{22} - b_{12} b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}. \end{aligned}$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C,$$

where adj is the adjoint of C.

adjoint