

D.01 Euler's formulas

Euler's formula

Euler's formula is our bridge back-and-forth between trigonometric forms ($\cos \theta$ and $\sin \theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos \theta + j \sin \theta. \quad (1)$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos \theta - j \sin \theta \quad (2a)$$

$$\cos \theta = \operatorname{Re} (e^{j\theta}) \quad (2b)$$

$$= \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \quad (2c)$$

$$\sin \theta = \operatorname{Im} (e^{j\theta}) \quad (2d)$$

$$= \frac{1}{j2} (e^{j\theta} - e^{-j\theta}). \quad (2e)$$

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